## ? Test yourself

1 Define:
   a the **focal point of a converging lens**
   b the **focal length of a diverging lens**.
2 Explain what is meant by:
   a a **real image** formed by a lens
   b a **virtual image** formed by a lens.
3 Explain why a real image can be projected on a screen but a virtual image cannot.
4 A plane mirror appears to reverse left and right. Does a lens do the same? Explain your answer.
5 A converging lens has a focal length of 6.0 cm. Determine the distance x.
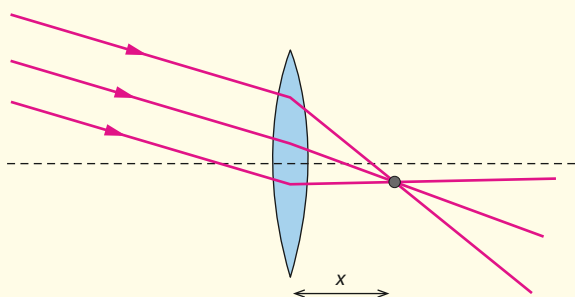
6 The diagram below shows the real image of an object in a converging lens. Copy the diagram and complete the rays drawn.
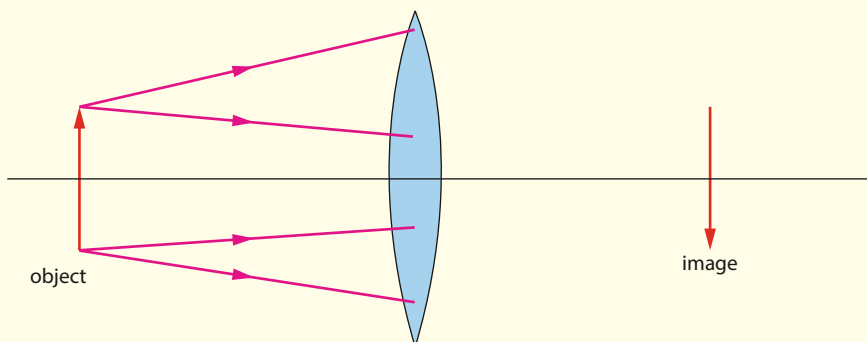
7 An object 2.0 cm tall is placed in front of a converging lens of focal length 10 cm. Using ray diagrams, construct the image when the object is at a distance of:
   a 20 cm   b 10 cm   c 5.0 cm.
   Confirm your ray diagrams by using the lens equation.
8 Using a ray diagram, determine the image characteristics of an object of height 2.5 cm that is placed 8.0 cm in front of a converging lens of focal length 6.0 cm. Confirm your ray diagram by using the lens equation.

9 Using a ray diagram, determine the image characteristics of an object of height 4.0 cm that is placed 6.0 cm in front of a converging lens of focal length 8.0 cm. Confirm your ray diagram by using the lens equation.
10 A converging lens of focal length 4.5 cm produces a real image that is the same size as the object. Determine the distance of the object from the lens.
11 Consider a converging lens of focal length 5.00 cm. An object of length 2.24 cm is placed in front of it, as shown below (not to scale), so that the middle of the object is on the principal axis. By drawing appropriate rays, determine the image in the lens. Is the angle the image makes with the principal axis the same as that for the object?
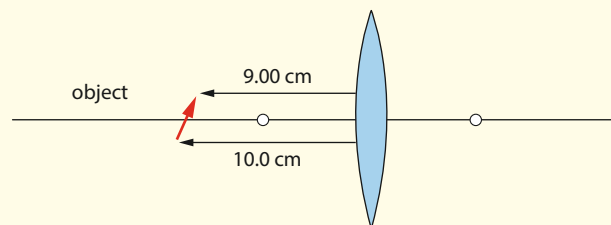

12 A student finds the position of the image created by a converging lens for various positions of the object. She constructs a table of object and image distances.

u/cm$\pm$0.1 cm	12.0	16.0	20.0	24.0	28.0
v/cm$\pm$0.1 cm	60.0	27.2	19.9	17.5	16.8

   a Explain how these data can be used to determine the focal length of the lens.
   b Determine the focal length, including the uncertainty in its value.

13 An object is placed in front of a converging lens which rests on a plane mirror, as shown below. The object is moved until the image is formed exactly at the position of the object itself. Draw rays from the object to form the image in this case. Explain how the focal length of the lens can be determined from this arrangement.
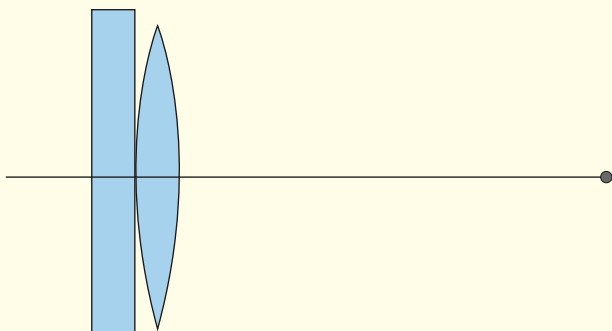
14 A converging lens has a focal length of 15 cm. An object is placed 20 cm from the lens.
   a Determine the image (i.e. its position and whether it is real or virtual, and upright or inverted) and find the magnification.
   b Draw a ray diagram to confirm your results.

15 An object is 5.0 m from a screen. A converging lens of focal length 60 cm is placed between the object and the screen so that an image of the object is formed on the screen.
   a Determine the distances from the screen where the lens could be placed for this to happen.
   b Determine which choice results in the larger image.

16 An object is placed 12 cm in front of a diverging lens of focal length 4.0 cm. Determine the properties of the image algebraically and with a ray diagram.

17 Two very thin lenses of focal lengths f_1 and f_2 are placed in contact. Show that the focal length of the two-lens system is given by $f = \dfrac{f_1 f_2}{f_1 + f_2}$.

18 Two converging lenses, each of focal length 10.0 cm, are 4.00 cm apart. Find the focal length of this lens combination.

19 An object is viewed through a system of two converging lenses, L_1 and L_2 (L_2 to the right of L_1). L_1 has a focal length of 15.0 cm and L_2 has a focal length of 2.00 cm. The distance between the lenses is 25.0 cm and the distance between the object (placed to the left of L_1) and L_1 is 40.0 cm. Determine:
   a the position of the image
   b the magnification of the image
   c the orientation of the image.

20 An object is viewed through a system of two lenses, L_1 and L_2 (L_2 to the right of L_1). L_1 is converging and has a focal length of 35.0 cm; L_2 is diverging and has a focal length of 20.0 cm. The distance between the lenses is 25.0 cm and the distance between the object (placed to the left of L_1) and L_1 is 30.0 cm. Determine:
   a the position of the image
   b the magnification of the image
   c the orientation of the image.

21 a An object is placed 4.0 cm in front of a concave mirror of focal length 12 cm. Determine the properties of the image.
   b Repeat part a when the concave mirror is replaced by a convex mirror of the same focal length.
   c In each case draw a ray diagram to show the construction of the image.

22 An object that is 15 mm high is placed 12 cm in front of a mirror. An upright image that is 30 mm high is formed by the mirror. Determine the focal length of the mirror and whether the mirror is concave or convex.

23 a Describe the two main lens aberrations and indicate how these can be corrected.

b In an attempt to understand the distortion caused by spherical aberration, a student considers the following model. She places an object of height 4.00 cm a distance of 8.00 cm from a converging lens. One end of the object is 1.00 cm below the principal axis and the other 3.00 cm above. She assumes that rays leaving the bottom of the object will have a focal length of 4.00 cm and the rays from the top a focal length of 3.50 cm (see diagram below).

  i Under these assumptions, draw rays from the bottom and top of the object to locate the image.

  ii Draw the image again by using a 4.00 cm focal length for all rays, and compare.
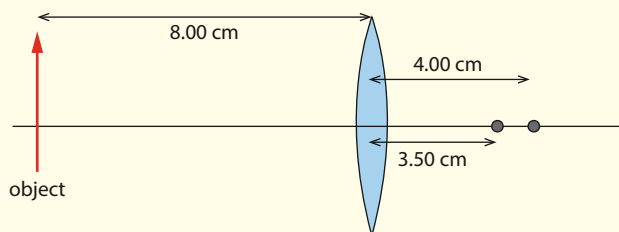
24 An object is placed in front and to the left of a converging lens, and a real image is formed on the other side of the lens. The distance of the object from the left focal point is x and the distance of the image from the right focal point is y. Show that $xy = f^2$.

25 A converging lens of focal length 10.0 cm is used as a magnifying glass. An object whose size is 1.6 mm is placed at some distance from the lens so that a virtual image is formed 25 cm in front of the lens.

a Calculate the distance between the object and the lens.

b Suggest where the object should be placed for the image to form at infinity.

c Find the angular size of the image at infinity.

26 Angular magnification, for a magnifying glass, is defined as $M = \dfrac{\theta'}{\theta}$.

a By drawing suitable diagrams, show the angles that are entered into this formula.

b A simple magnifying glass produces an image at the near point. Explain what is meant by 'near point'.

c Show that when a simple magnifying glass produces an image at the near point, the magnification is given by $M = 1 + \dfrac{25}{f}$, where f is the focal length of the lens in cm.

27 The normal human eye can distinguish two objects 0.12 mm apart when they are placed at the near point. A simple magnifying glass of focal length 5.00 cm is used to view images at the near point. Determine how close the objects can be and still be distinguished.

C2 Imaging instrumentation

We owe much of our knowledge about the natural world to optical instruments based on mirrors and lenses. These have enabled the observation of very distant objects through telescopes and very small objects through microscopes. We have already seen how a single converging lens can produce an enlarged upright image of an object placed closer to the lens than the focal length, thus acting as a magnifying glass. The apparent size of an object depends on the size of the image that is formed on the retina. In turn, this size depends on the angle subtended by the object at the eye. This is why we bring a small object closer to the eye in order to view it – the angle subtended at the eye by the object increases.

C2.1 The optical compound microscope

A **compound microscope** (Figure **C.34**) consists of two converging lenses. It is used to see enlarged images of very small objects. The object (of height h) is placed at a distance from the first lens (the **objective**)

28 The objective of a microscope has a focal length of 0.80 cm and the eyepiece has a focal length of 4.0 cm. An object is placed 1.50 cm from the objective. The final image is formed at the near point of the eye (25 cm).
   a Calculate the distance of the image from the objective.
   b Calculate the distance from the eyepiece lens of the image in **a**.
   c Calculate the angular magnification of the microscope.

29 In a compound microscope the objective focal length is 20 mm and the eyepiece focal length is 80 mm. An object is placed 25 mm from the objective. The final virtual image is formed 35 cm from the eyepiece.
   **a** Calculate the distance of the image from the objective.
   b Calculate the distance from the eyepiece lens of the image in **a**.
   c Calculate the angular magnification of the microscope.

30 The diagram below illustrates a compound microscope. Copy the diagram and draw rays in order to construct the final image.
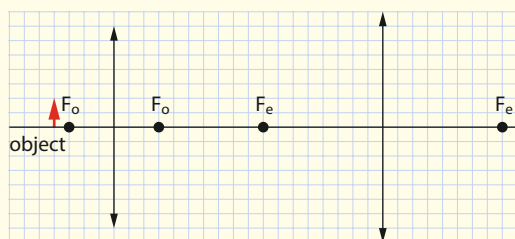
31 A compound microscope forms the final image at a distance of 25 cm from the eyepiece. The eye is very close to the eyepiece. The objective focal length is 24 mm and the object is placed 30 mm from the objective. The angular magnification of the microscope is 30. Determine the focal length of the eyepiece.

32 The diagram below shows rays from a distant object arriving at a refracting telescope. Copy the diagram and complete the rays to show the formation of the final image at infinity.
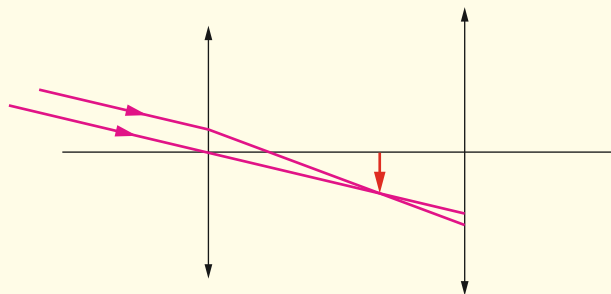
33 An astronomical telescope is in normal adjustment.
   a State what is meant by this statement.
   b The angular magnification of the telescope is 14 and the focal length of the objective is 2.0 m. Calculate the focal length of the eyepiece.

34 The Moon is at a distance of 3.8×10^8 m from the Earth and its diameter is 3.5×10^6 m.
   a Show that the angle subtended by the diameter of the Moon at the eye of an observer on the Earth is 0.0092 rad.
   b A telescope objective lens has a focal length of 3.6 m and an eyepiece focal length of 0.12 m. Calculate the angular diameter of the image of the Moon formed by this telescope.

35 A telescope consists of an objective, which is a converging lens of focal length 80.0 cm, and the eyepiece of has a focal length 20.0 cm. The object is very far from the objective (effectively an infinite distance away) and the image is formed at infinity.
   **a** Calculate the angular magnification of this telescope.
   b The telescope is used to view a building of height 65.0 m a distance of 2.50 km away. Calculate the angular size of the final image.

36 A refracting telescope has an eyepiece of focal length 3.0 cm and an objective of focal length 67.0 cm.
   a Calculate the magnification of the telescope.
   **b** State the length of the telescope. (Assume that the final image is produced at infinity.)

42 Calculate the speed of light in the core of an optical fibre of refractive index 1.45.

43 a State what is meant by **total internal reflection**.

   b Define **critical angle**.

   c Explain why total internal reflection can only occur for a ray travelling from a high- to a low-refractive-index medium and not the other way around.

44 The refractive indices of the core and the cladding of an optical fibre are 1.50 and 1.46, respectively. Calculate the critical angle at the core–cladding boundary.

45 In an optical fibre, n_1 and n_2 are the refractive indices of the core and the cladding, respectively (so $n_1 > n_2$).
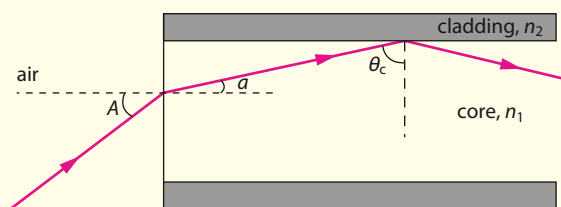

   a Show that the cosine of the critical angle is given by
   $$\cos\theta_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

   b Hence show that the maximum angle of incidence A from air into the core that will result in the ray being totally internally reflected is given by
   $$A = \arcsin\sqrt{n_1^2 - n_2^2}$$

   c Calculate the acceptance angle of an optical fibre with a core refractive index of 1.50 and cladding refractive index of 1.40.

46 Calculate the acceptance angle of an optical fibre with core and cladding refractive indices equal to 1.52 and 1.44, respectively.

47 The refractive index of the cladding of an optical fibre is 1.42. Determine the refractive index of the core such that any ray entering the fibre gets totally internally reflected.

48 State **one** crucial property of the glass used in the core of an optical fibre.

49 a State what is meant by **dispersion** in the context of optical fibres.

   b Distinguish between waveguide and material dispersion.

50 An optical fibre has a length of 8.00 km. The core of the optical fibre has a refractive index of 1.52 and the core–cladding critical angle is 82°.

   a Calculate the speed of light in the core.

   b Calculate the minimum and maximum times taken for a ray of light to travel down the length of the fibre.

51 The pulse shown below is input into a multimode optical fibre. Suggest the shape of the output pulse after it has travelled a long distance down the fibre.
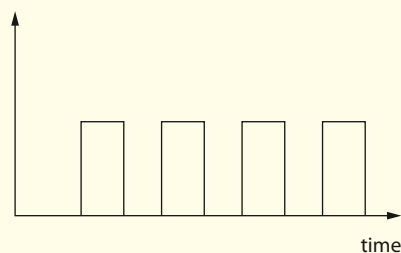
52 a Distinguish between **monomode** and **multimode** optical fibres.

   b Discuss the effect of reducing the fibre core diameter on the bandwidth that can be transmitted by the fibre.

53 List **three** advantages of optical fibres in communications.

54 State the main cause of attenuation in an optical fibre.

55 Two amplifiers of gain G_1 and G_2 (in dB) amplify a signal, as shown below. Calculate the overall gain produced by the two amplifiers.
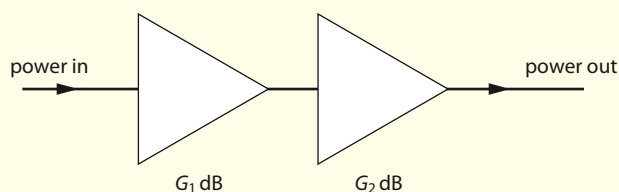

56 A signal of power 4.60 mW is attenuated to 3.20 mW. Calculate the power loss in decibels.

57 A signal of power 8.40 mW is attenuated to 5.10 mW after travelling 25 km in a cable. Calculate the attenuation per unit length of the cable.

58 A coaxial cable has a specific attenuation of 12 dB km^{-1}. The signal must be amplified when the power of the signal falls to 70% of the input power. Determine the distance after which the signal must be amplified.

59 A signal is input into an amplifier of gain +15 dB. The signal then travels along a cable, where it suffers a power loss of 12 dB. Calculate the ratio of the output power to the input power.

+15 dB   cable   −12 dB
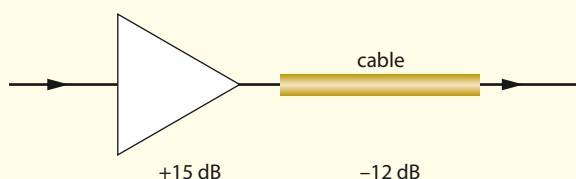
60 A signal is input into an amplifier of gain +7.0 dB. The signal then travels along a cable, where it suffers a power loss of 10 dB, and is then amplified again by an amplifier of gain +3.0 dB. Calculate the ratio of the output power to the input power.

+7 dB   cable −10 dB   +3 dB
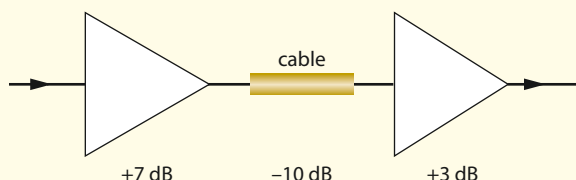
61 In the arrangement shown below, the output power is twice the input power. Calculate the required gain G of the amplifier.

amplifier

cable   cable

−12 dB G dB   −6.0 dB
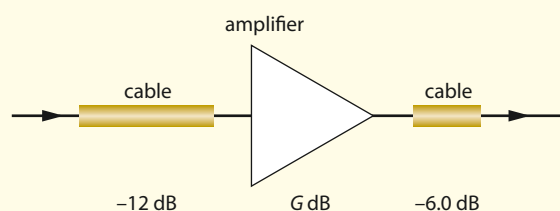
62 a Sketch a graph (no numbers are required on the axes) to illustrate the variation with wavelength of the specific attenuation in an optical fibre.
   b Explain why infrared wavelengths are preferred in optical fibre transmission.

C4 Medical imaging (HL)

This section introduces the use of X–rays and ultrasound in medical imaging. Other imaging techniques, including PET scans and a method based on nuclear magnetic resonance, are also discussed.

C4.1 X-ray imaging

X–rays are electromagnetic radiation with a wavelength around 10^{-10} m. X–rays for medical use are produced in X–ray tubes, in which electrons that have been accelerated to high energies by high potential differences collide with a metal target. As a result of the deceleration suffered by the electrons during the collisions and transitions between energy levels in the target atoms, X–rays are emitted (see Figure **C.51**). This was the first radiation to be used for medical imaging. Typical hospital X–ray machines operate at voltages of around 15–30 kV for a mammogram or 50–150 kV for a chest X–ray.

X–rays travelling through a medium suffer energy loss, referred to as **attenuation**. The dominant mechanism for this is the photoelectric effect: X–ray photons are absorbed by electrons in the medium and energy is transferred to the electrons. The effect is strongly dependent on the atomic number of the atoms of the medium. There is a substantial difference between the atomic numbers of the elements present in bone ($Z = 14$) and soft tissue ($Z = 7$), and bone absorbs X–rays more strongly than soft tissue. Hence, an X–ray image will show a **contrast** between bone and soft tissue.

Learning objectives

- Understand the use of X–rays in medical imaging.
- Understand the use of ultrasound in medical imaging.
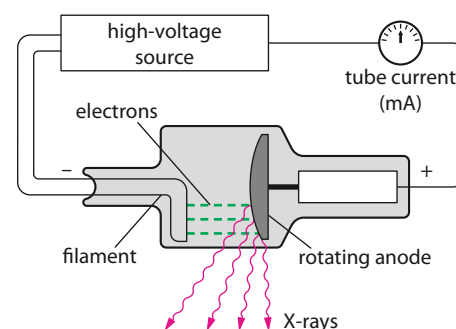- Understand magnetic resonance imaging in medicine.

Figure C.51 Schematic diagram of an X-ray tube.

Exam-style questions

1 **a** State what is meant by the **focal length** of a converging lens. **[2]**

    b In order to view the detail on an ancient coin, an art dealer holds a converging lens 2.0 cm above the coin. A virtual upright image of the coin is formed with a magnification of 5.0. Calculate the focal length of the lens **[3]**

    c Determine where the object should be placed so that the magnification produced is as large as possible. **[3]**

2 The diagram below shows an object placed in front of a converging lens. The lens forms an image of the object. The diagram also shows a ray R from the object.
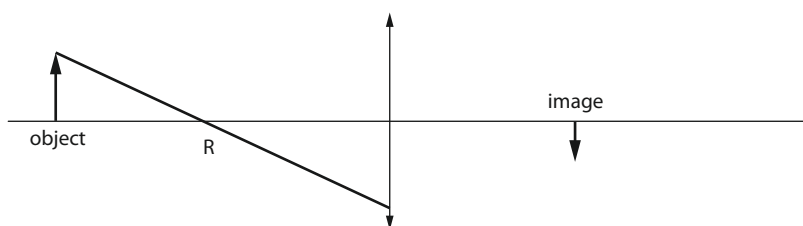

    a On a copy of the diagram, extend the ray R to show how it refracts in the lens. **[1]**

    b Draw an appropriate ray to locate the focal points of the lens. **[2]**

    c State and explain whether the image formed is real or virtual. **[2]**

    d The upper half of the lens is covered with opaque paper. State and explain the effect of this, if any, on the image. **[2]**

    e A converging lens of focal length 4.0 cm is used as a magnifying glass in order to view an object of length 5.0 mm placed at right angles to the axis of the lens. The image is formed 25 cm from the eye, which is placed very close to the lens. Determine:
        i the distance of the object from the lens **[2]**
        ii the length of the image **[2]**
        iii the angle subtended by the image at the eye. **[1]**

3 A compound microscope consists of an objective lens of focal length 15 mm and an eyepiece lens of focal length 60 mm. The final image of an object placed 20 mm from the objective is formed 25 cm from the eyepiece lens.

    a Determine:
        i the distance of the image formed by the objective from the objective lens **[2]**
        ii the distance of the image in **i** from the eyepiece lens. **[2]**

    b **i** State what is meant by the **angular magnification** of a microscope. **[2]**
        ii Determine the angular magnification of the microscope. **[2]**

    c The object has a length of 8.0 mm and is placed at right angles to the axis of the microscope. Calculate:
        i the length of the final image **[1]**
        ii the angle subtended by the final image at the eyepiece lens. **[1]**

4 An astronomical refracting telescope consists of two converging lenses.

a Suggest a reason why the diameter of the objective lens of a telescope should be large. [1]

b An astronomical telescope is used to view the Sun. The diagram below (not to scale) shows the formation of the intermediate image of the Sun. On a copy of the diagram, draw lines to show the formation of the real image of the Sun in the eyepiece. [2]
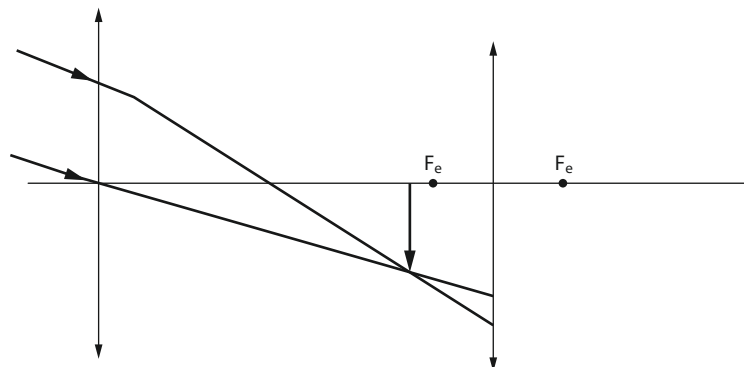
c The focal length of the objective is 1.00 m and that of the eyepiece is 0.10 m. Calculate the distance of the image in **b** from the eyepiece, given that the eyepiece forms a **real** image 0.455 m from the eyepiece. [2]

d The rays of the Sun make an angle of 0.055 rad with the axis of the objective. Determine the size of the image in the eyepiece. [3]

5 a An object of height 3.0 cm is placed 8.0 cm in front of a concave spherical mirror of focal length 24 cm. Calculate:
   i the position of the image [1]
   ii the height of the image. [2]

b Draw a ray diagram to illustrate your answers to **i** and **ii**. [3]

c An object is placed in front of a mirror. An upright image half the height of the object is formed behind the mirror. The distance between the object and the image is 120 cm. Calculate the focal length of the mirror. [3]

d Telescopes use mirrors rather than lenses.
   i Outline **two** advantages of mirrors over lenses in a telescope. [4]
   ii State **one** advantage of parabolic mirrors over spherical mirrors in a telescope. [1]

6 The diagram below (not to scale) shows a Cassegrain-type reflecting telescope. The small arrow shows the image of a planetary feature that would be formed by the concave mirror in the absence of the small convex mirror. The actual feature subtends an angle $\alpha = 1.50 \times 10^{-4}$ rad at the concave mirror. The focal length of the concave mirror is 10.0 m.
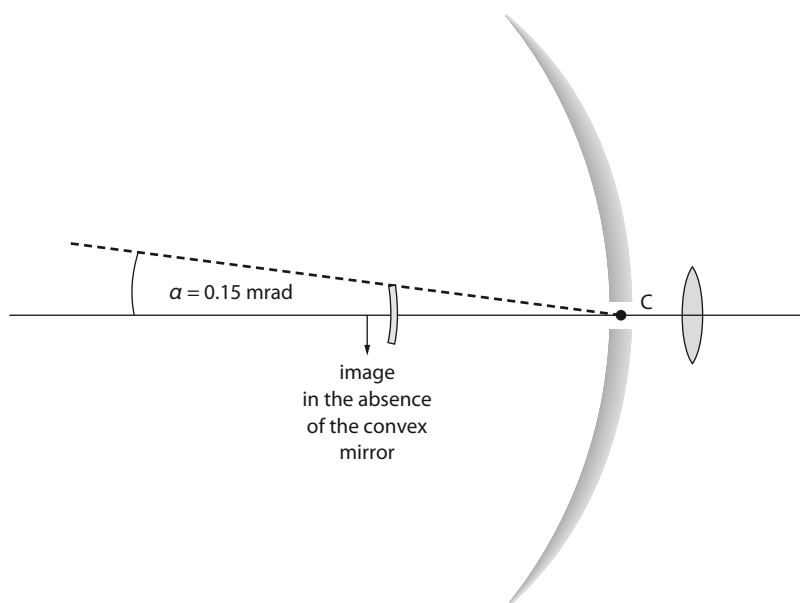
a Calculate the length of the image shown here. **[2]**

b This image serves in turn as the object for the small convex mirror, which produces a real image at C. The concave and convex mirrors are separated by 9.00 m. Calculate:
  i the focal length of the convex mirror **[2]**
  ii the magnification of the convex mirror **[1]**
  iii the height of the image at C. **[1]**

c The image at C is viewed through a converging lens of focal length 12 cm, forming a virtual image very far away.
  i Calculate the angle subtended at the converging lens by the image at C. **[2]**
  ii Hence calculate the overall angular magnification of this system. **[2]**

7 a State what is meant by a **lens aberration**. **[1]**

b **i** **Spherical** and **chromatic aberration** are two common types of lens aberration. Describe what is meant by each. **[4]**
  ii Describe **one** way in which each of the aberrations in **i** may be reduced. **[2]**

8 Two objects, A and B, each of height 5.0 cm, are placed in front of a concave mirror of focal length 24 cm. The distances of the objects A and B from the mirror are, respectively, 40 cm and 30 cm.

a Calculate:
  i the distance between the images of A and B **[3]**
  ii the difference in heights of the images of A and B. **[3]**

b A rod of length 10 cm is placed in front of the concave mirror such that it is parallel to the principal axis of the mirror and 5.0 cm to the side of it. The front of the rod is 30 cm from the mirror. Use your answer in **a** to determine whether the image of the rod:
  i has the same length as the rod itself **[2]**
  ii is parallel to the principal axis. **[2]**

9 The diagram shows a ray of light entering the core of an optical fibre from air. The core has a refractive index of 1.58 and the cladding a refractive index of 1.45.

cladding

A
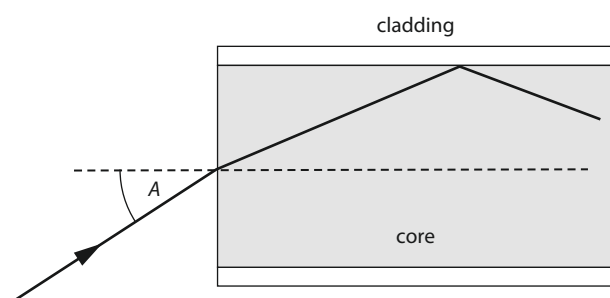
core

a Determine:
   i the critical angle at the core–cladding boundary [2]
   ii the largest angle of incidence A such that the ray will undergo total internal reflection at the core–cladding boundary. [2]

b Distinguish between **waveguide dispersion** and **material dispersion** in an optical fibre. [3]

c Outline how **i** waveguide dispersion and **ii** material dispersion may be reduced. [4]

d The power of a signal input into an optical fibre is 25.0 mW. The attenuation per unit length for this fibre is 3.50 dB km^{-1}. The signal power must not fall below 15.0 μW.
   i State one source of attenuation in an optical fibre. [1]
   ii Determine the distance after which the signal must be amplified. [3]

HL 10 a i State what is meant by **ultrasound**. [1]
   ii Ultrasound and X-rays are equally capable of imaging parts of the body. Suggest why ultrasound would be the preferred method of imaging. [1]

b The impedance of air is $Z_{air} = 410$ kg m^{-2} s^{-1} and that of soft tissue is $Z_{tissue} = 1.8 \times 10^6$ kg m^{-2} s^{-1}.

The fraction of the intensity that gets reflected back from the air–tissue boundary is $\dfrac{I_t}{I_0} = \dfrac{(Z_{air} - Z_{tissue})^2}{(Z_{air} + Z_{tissue})^2}$.
   i Calculate this fraction. [1]
   ii Comment on the answer to **i**, suggesting a solution to the problem it reveals. [3]

c A pulse of ultrasound is reflected from the boundary of an organ 6.5 ms after it is emitted. The region between the surface of the skin, where the pulse originates, and the organ is filled with tissue in which the speed of sound is 1500 m s^{-1}. Estimate the distance of the organ boundary from the surface of the skin. [2]

HL 11 a State what is meant by **half-value thickness** (HVT). [1]

b The half-value thickness of soft tissue for X-rays of a given energy is 4.10 mm.
   i After a distance x in soft tissue, the fraction of the incident intensity of X-rays that gets transmitted is 0.650. Determine this distance. [3]
   ii State and explain the effect, if any, on the answer to **i** if X-rays with a larger half-value thickness were to be used. [2]

c Outline how, in X-ray imaging, the following are achieved:
   i reduction of the blurring in the image [2]
   ii reduction of the exposure time. [2]

HL 12 Outline the technique of **magnetic resonance imaging**. [6]