Answers to test yourself questions

Option C

C1 Introduction to imaging

1 **a** The focal point of a converging lens is that point on the principal axis where a ray parallel to the principal axis refracts through, after passage through the lens.

b The focal length is the distance of the focal point from the middle of the lens. In the lens equation this is taken to be a negative number.

2 **a** A real image is an image formed by actual rays of light which have refracted through a lens.

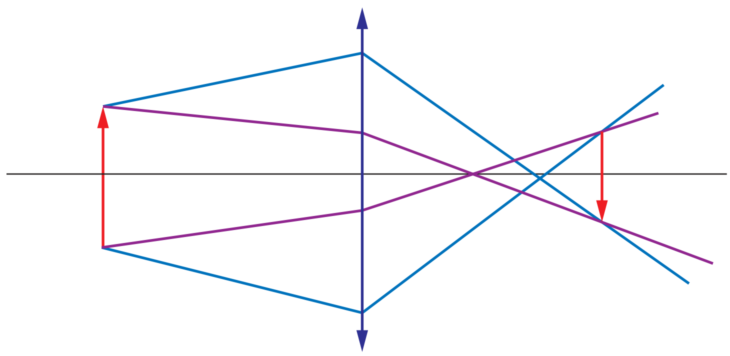
b A virtual image is formed not by actual rays but by the intersection of their mathematical extensions.

3 If a screen is placed at the position of a real image the actual rays of light that go through that image will be reflected off the screen and so the image will be seen on the screen. In the case of a virtual image, placing a screen at the position of the image reveals nothing as there are no rays of light to reflect off the screen.

4 No one can explain things better than Feynman and this case is no exception. Search for the YouTube video "Feynman: FUN TO IMAGINE 6: The Mirror" where Feynman explains the apparent left-right case for the mirror. Then try to see what happens with a lens.

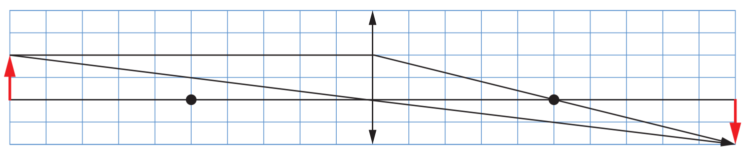
5 The distance is the focal length so 6.0 cm.

6 See graph shown.



7 **a** The diagrams use a vertical scale of 1 cm per line and a horizontal scale of 2 cm per line.
$u = 20$ cm:
The formula gives:

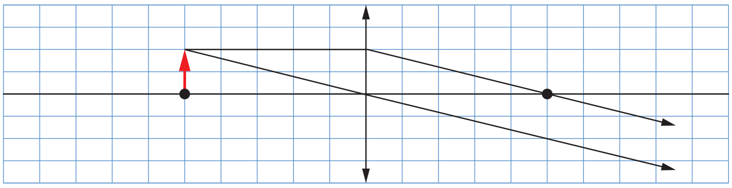
$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20} \Rightarrow v = +20$ cm. Further $M = -\frac{v}{u} = -\frac{20}{20} = -1$. So the image is real (positive v), 20 cm on the other side of the lens, and the image is inverted (negative M) and has height 2 cm ($|M| = 1$). This is what the ray diagram below also gives.



b $u = 10$ cm:

The formula gives:

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{10} - \frac{1}{10} = 0 \Rightarrow v = \infty$. The image is formed at infinity. This is what the ray diagram gives.

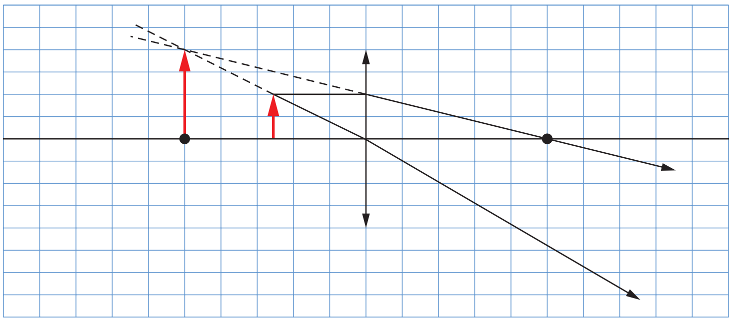


Rays do not meet even when they are extended. Image is said to "form at infinity".

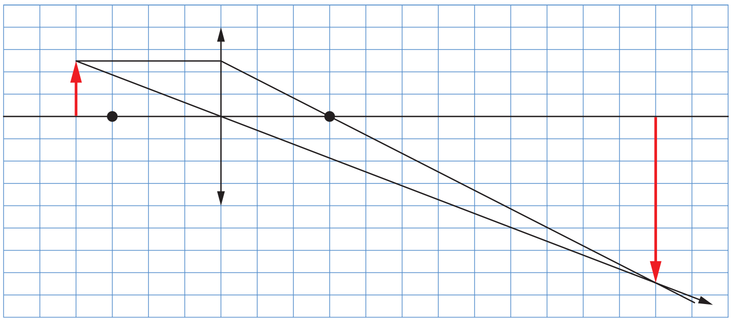
c $u = 5.0$ cm:

The formula gives:

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{10} - \frac{1}{5.0} = -\frac{1}{10} \Rightarrow v = -10$ cm. Further $M = -\frac{v}{u} = -\frac{-10}{5.0} = +2$. So the image is virtual (negative v), 10 cm on the same side of the lens, and the image is upright (positive M) and has height twice as large (i.e. 4 cm) (because $|M| = 2$). This is what the ray diagram below also gives.



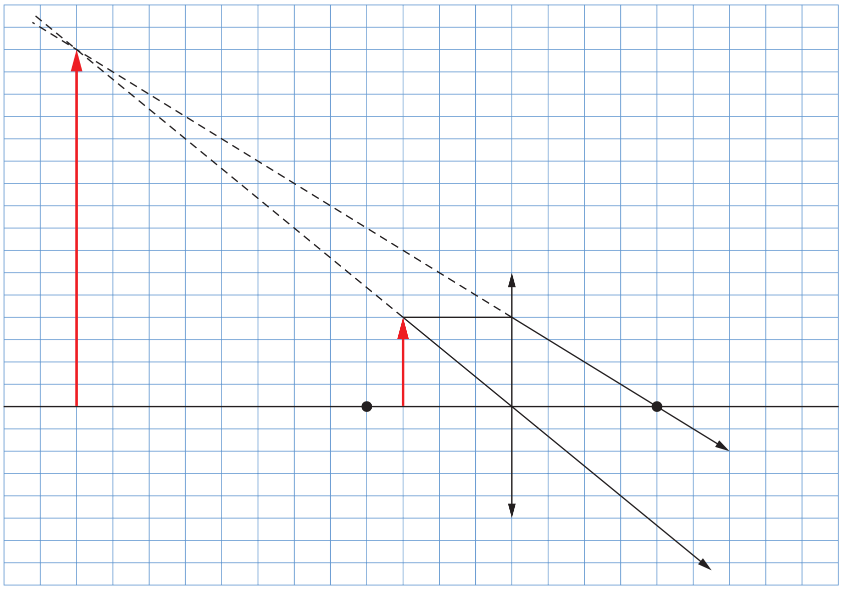
8 The diagram uses a vertical scale of 1 cm per line and a horizontal scale of 2 cm per line.



The formula gives:

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{6.0} - \frac{1}{8.0} \Rightarrow v = +24$ cm. Further $M = -\frac{v}{u} = -\frac{24}{8.0} = -3$. So the image is real (negative v), 24 cm on the other side of the lens, and the image is inverted (negative M) and has height 3 as large (i.e. 7.5 cm) (because $|M| = 3$). This is what the ray diagram above also gives.

9 See graph shown.



The diagram uses a vertical scale of 1 cm per line and a horizontal scale of 2 cm per line.

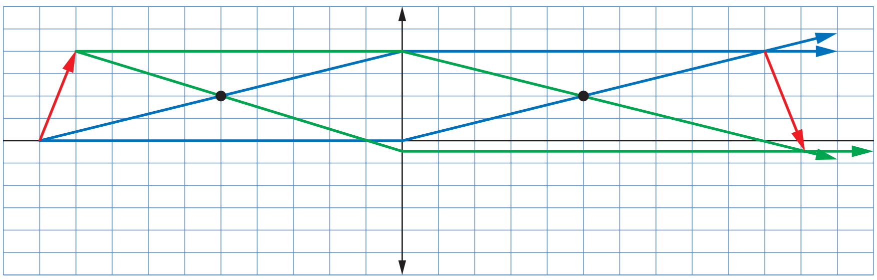
The formula gives:

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{8.0} - \frac{1}{6.0} = -\frac{1}{24} \Rightarrow v = -24$ cm. Further $M = -\frac{v}{u} = -\frac{-24}{6.0} = +4$. So the image is virtual (negative v), 24 cm on the same side of the lens, and the image is upright (positive M) and has height 4 as large (i.e. 16 cm) (because $|M| = 4$). This is what the ray diagram below also gives.

10 We know that $v > 0$ (real image) and $M = -1 = -\frac{v}{u}$ (same size). Hence $v = u$ and so

$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{2}{u} = \frac{1}{f} \Rightarrow u = 2f = 9.0$ cm.

11 See the diagram that follows. The rays from the top of the object have been drawn green and those from the bottom blue for the sake of clarity.



The top of the image will be formed at a distance from the lens given by:

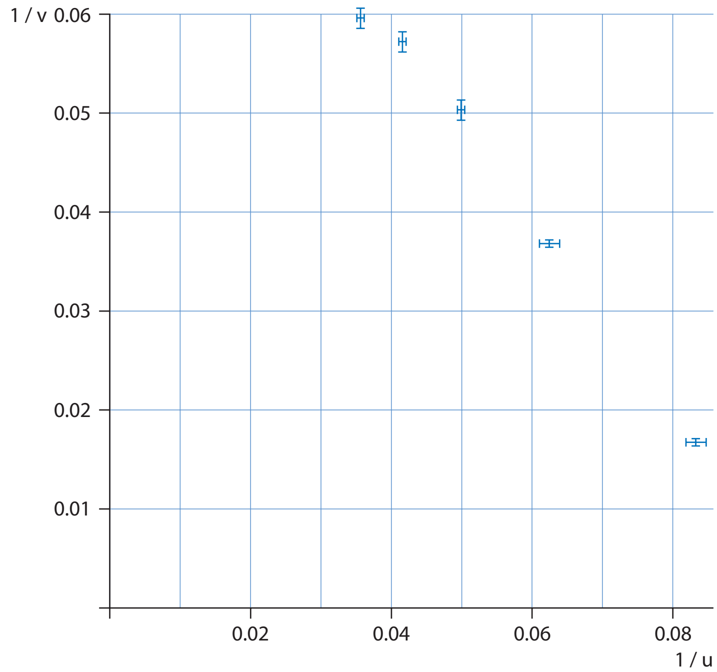
$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{5.0} - \frac{1}{9.0} \Rightarrow v = 11.25$ cm and the bottom at a distance of

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{5.0} - \frac{1}{10.0} \Rightarrow v = 10.0$ cm.

The angle the object makes with the horizontal is $\tan^{-1}\frac{4}{1} \approx 76°$. The image makes an angle given by $\tan^{-1}\frac{4.5}{1.25} \approx 75°$ so it is slightly smaller.

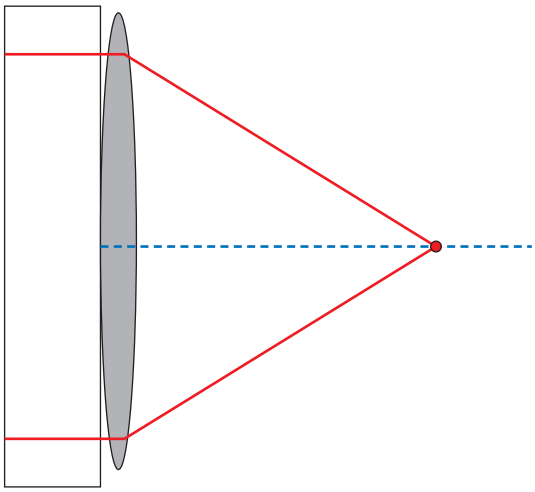
12 **a** Since we know that $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ we should plot $\frac{1}{u}$ versus $\frac{1}{v}$. We expect a straight line with gradient equal to −1 and equal vertical and horizontal intercepts equal to $\frac{1}{f}$.

b The graph shows the data points and the (small) error bars.



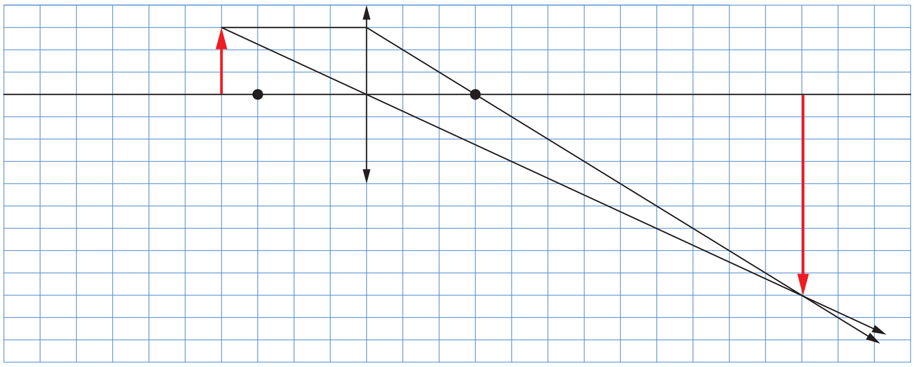
The line of best fit is $\frac{1}{v} = 0.096 - \frac{0.95}{u}$. The intercepts are 0.096 and $\frac{0.096}{0.95}$ giving focal lengths $\frac{1}{f} = 0.096 \Rightarrow f = 10.42$ cm and $\frac{1}{f} = \frac{0.096}{0.95} \Rightarrow f = 9.896$ cm. So approximately, $f = 10.2 \pm 0.3$ cm.

13 From the diagram it should be clear that rays must hit the mirror at right angles if they are to return to the position of the object. This means that the distance of the object from the lens when the object and image coincide is the focal length.



14 a $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{15}-\frac{1}{20}\Rightarrow v=+60$ cm. Further $M=-\frac{v}{u}=-\frac{60}{20}=-3$. So the image is real (positive v), 60 cm on the other side of the lens, and the image is inverted (negative M) and has height 3 times as large (because $|M|=3$). This is what the ray diagram below also gives.

b See graph shown.

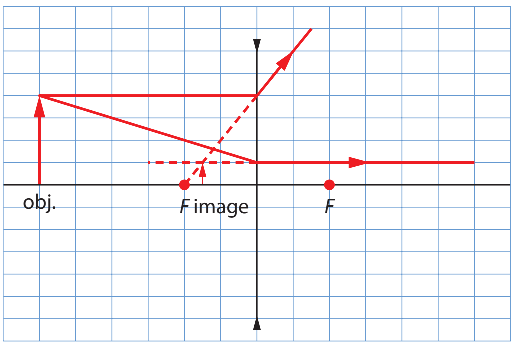


15 a We must have that $u+v=5$ and $\frac{1}{u}+\frac{1}{v}=\frac{1}{0.6}$ where distances are in meters. Then $v=50-u$ and so $\frac{1}{5-v}+\frac{1}{v}=\frac{1}{0.6}$. This gives $v^2-5v+3=0$ with solutions $v=4.30$ m or $v=0.70$ m.

b In the first case the magnification is $M=-\frac{4.30}{0.70}=-6.1$ and in the second $M=-\frac{0.70}{4.30}=-0.16$ so the magnification is larger (in magnitude) in the first case.

16 We use $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ with $f=-4.0$ cm. Hence $\frac{1}{v}=-\frac{1}{4.0}-\frac{1}{12}=-\frac{1}{3.0}$. Hence $v=-3.0$ cm and $M=-\frac{-3.0}{12}=+\frac{1}{4.0}$.

The image is virtual ($v<0$), upright and smaller by a factor of 4 $\left(M=+\frac{1}{4.0}\right)$.



17 Let u be the distance of an object from the first lens. The lens creates an image a distance v from the lens which is given by $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ hence $\frac{1}{v}=\frac{1}{f_1}-\frac{1}{u}$. This image serves as the virtual object in the second lens. Because the lenses are thin the distance of this virtual object is also v. Hence the final image is formed at distance of $-\left(\frac{1}{f_1}-\frac{1}{u}\right)+\frac{1}{v_2}=\frac{1}{f_2}$ (the minus sign in front of the first term is because the object is virtual) and so $\frac{1}{v_2}=\left(\frac{1}{f_2}+\frac{1}{f_1}\right)-\frac{1}{u}$. This is what we would have obtained if the inverse focal length of the combination were $\frac{1}{f}=\frac{1}{f_2}+\frac{1}{f_1}$. Hence $f=\frac{f_1f_2}{f_1+f_2}$.

18 Using $f = \dfrac{f_1 f_2}{f_1 + f_2}$ we find $f = \dfrac{10.0 \quad 4.0}{14.0} = 2.86$ cm.

19 **a** The image in the first lens is found from: $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{10} \Rightarrow \dfrac{1}{40.0} + \dfrac{1}{v} = \dfrac{1}{15.0} \Rightarrow v = 24.0$ cm. This means that the distance of the image from the second lens is 1.0 cm. This image now serves as the object for the second lens. So $\dfrac{1}{1.0} + \dfrac{1}{v} = \dfrac{1}{2.0} \Rightarrow v = -2.0$ cm. The final image is 2.0 cm to the left of the second lens.

b The overall magnification is the product of the individual lens magnifications i.e.

$$M = M_1 M_2 = \left(-\frac{24}{40}\right)\left(-\frac{-2.0}{1.0}\right) = -1.2.$$

c Since $M < 0$, the final image is inverted relative to the original object and is 1.2 times larger.

20 **a** The image in the first lens is found from: $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f} \Rightarrow \dfrac{1}{30.0} + \dfrac{1}{v} = \dfrac{1}{35.0} \Rightarrow v = -210$ cm. This means that the distance of the image from the second lens is 235 cm. This image now serves as the object for the second lens. So $\dfrac{1}{235} + \dfrac{1}{v} = -\dfrac{1}{20.0} \Rightarrow v = -18.4$ cm. The final image is 18.4 cm to the left of the second lens.

b The overall magnification is the product of the individual lens magnifications i.e.

$$M = M_1 M_2 = \left(-\frac{-210}{30}\right)\left(-\frac{-18.4}{235}\right) = +0.548 \approx 0.55.$$

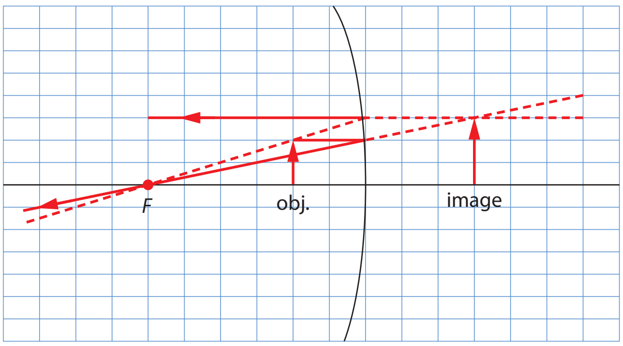
c Since $M > 0$, the final image is upright relative to the original object and is 0.55 times smaller.

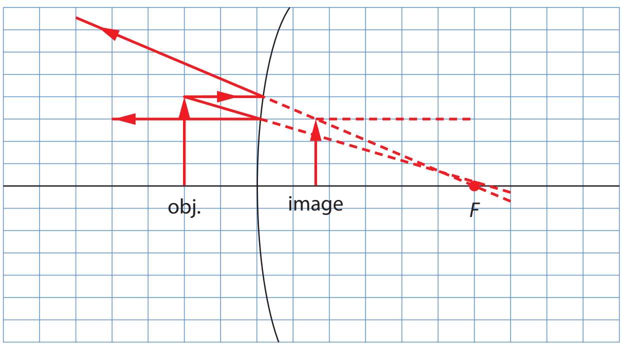
21 **a** The mirror formula gives $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f} \Rightarrow \dfrac{1}{v} = \dfrac{1}{12} - \dfrac{1}{4.0} \Rightarrow v = -6.0$ cm. The magnification is $M = -\dfrac{-6.0}{4.0} = +1.5$.

Therefore the image is virtual, formed on the same side of the mirror as the object, upright and 1.5 times taller.

b Now the mirror formula gives $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f} \Rightarrow \dfrac{1}{v} = -\dfrac{1}{12} - \dfrac{1}{4.0} \Rightarrow v = -3.0$ cm. The magnification is $M = -\dfrac{-3.0}{4.0} = +0.75$. Therefore the image is virtual, formed on the same side of the mirror as the object, upright and 0.75 times the object's height.

c See graphs shown.

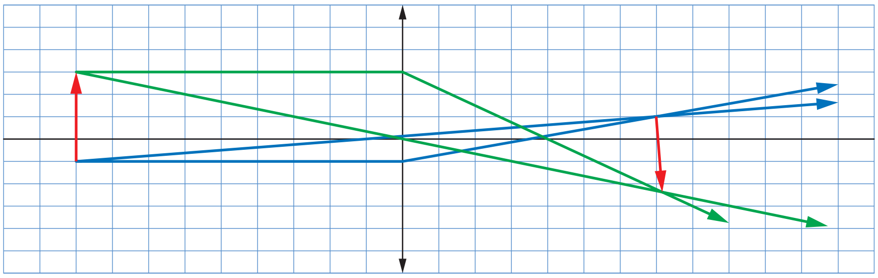




22 We are told that $M = +2.0$ (image is upright so $M > 0$) hence $2.0 = -\frac{v}{u} \Rightarrow v = -2.0u = -24$ cm. Hence from the mirror formula gives $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ we get $\frac{1}{f} = \frac{1}{12} - \frac{1}{24} \Rightarrow f = 24$ cm. Since the focal length is positive the mirror is concave.

23 **a** There are two main lens aberrations. In spherical aberration rays that are far from the principal axis have a different focal length than rays close to the principal axis. This results in images that are blurred and curved at the edges. In chromatic aberration, rays of different wavelength have slightly different focal lengths resulting in images that are blurred and coloured. Spherical aberration is reduced by only allowing rays close to the principal axis to enter the lens and chromatic aberration is reduced by combining the lens with a second diverging lens.

b **i** The diagram shows (under the simple conditions of this problem) that a different focal length (depending on the distance of the paraxial rays from the principal axis) creates an image that is curved at the edges.



ii The image drawn with one focal length would be straight.

24 We are told that $u = f + x$ and $v = f + y$. Then $\frac{1}{f+x} + \frac{1}{f+y} = \frac{1}{f} \Rightarrow \frac{(f+x)(f+y)}{f+x+f+y} = f$. Simplifying,

$$f^2 + fx + fy + xy = f(2f + x + y) = 2f^2 + fx + fy$$

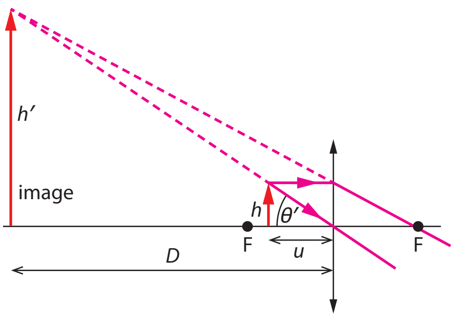
$$xy = f^2$$

25 **a** The image is virtual so $v = -25$ cm. $\frac{1}{u} + \frac{1}{v} = \frac{1}{10.0} \Rightarrow \frac{1}{u} - \frac{1}{25} = \frac{1}{10.0} \Rightarrow u = 7.143 \approx 7.1$ cm.

b At the focal point of the lens, 10 cm away.

c The angular magnification in this case is $M = \frac{25}{f} = \frac{25}{10} = 2.5$ and $M = \frac{\theta'}{\theta}$. Now $\theta \approx \frac{1.6 \times 10^{-3}}{0.25} = 0.0064$ rad and so $\theta' \approx 2.5 \times 0.0064 = 0.016$ rad.

26 **a** See graph shown.



b The nearest point to the eye where the eye can focus without straining.

c When the image is formed at the near point (25 cm away) we have that $v = -25$ cm.

Hence $\frac{1}{u} + \frac{1}{-25} = \frac{1}{f} \Rightarrow \frac{1}{u} = \frac{1}{f} + \frac{1}{25} = \frac{25+f}{25f} \Rightarrow u = \frac{25f}{25+f}$. The magnification is then

$$M = -\frac{v}{u} = -\frac{-25}{\frac{25f}{25+f}} = +\frac{25+f}{f} = 1 + \frac{25}{f}.$$

27 The magnification is $M = 1 + \frac{25}{f} = 1 + \frac{25}{5.0} = 6.0$. Hence if d is the distance of the two points we must have that $0.12 \times 10^{-3} = 6.0d$ and so $d = 2.0 \times 10^{-5}$ m.

C2 Imaging instrumentation

28 a The image in the objective is formed at a distance given by: $\frac{1}{1.50} + \frac{1}{v} = \frac{1}{0.80} \Rightarrow v = 1.71 \approx 1.7$ cm. The magnification of the objective is then $m_0 = -\frac{1.71}{1.50} = -1.14$.

b $\frac{1}{u} + \frac{1}{-25} = \frac{1}{4.0} \Rightarrow u = 3.45 \approx 3.4$ cm

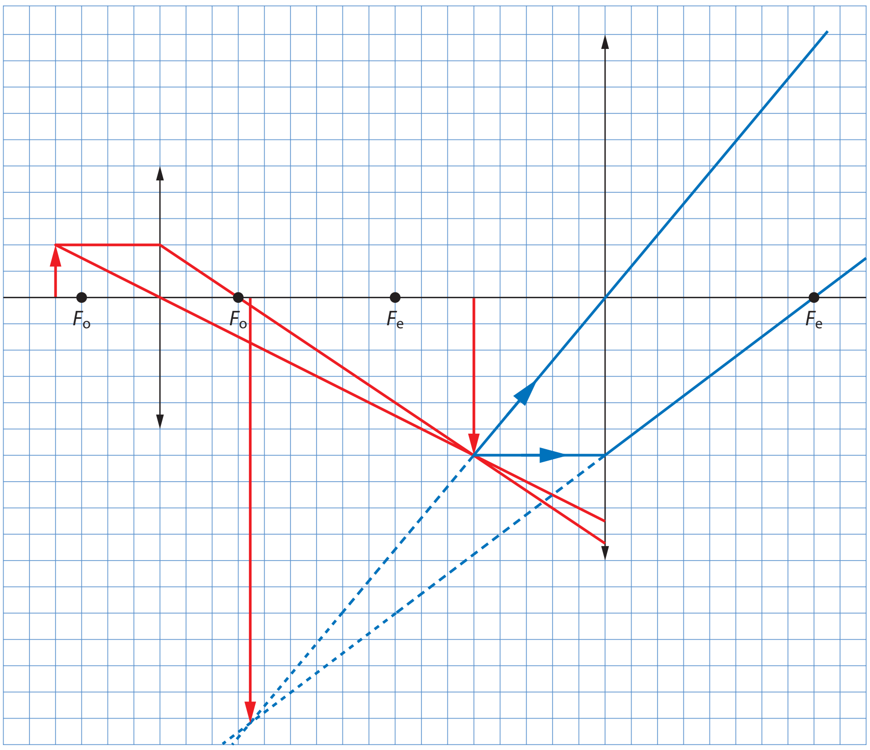
c The magnification of the eyepiece is $1 + \frac{25}{4.0} = 7.25$. The overall magnification is then $-1.14 \times 7.25 = -8.3$.

29 a From $\frac{1}{25} + \frac{1}{v_1} = \frac{1}{20}$ we get $v_1 = 100$ mm.

b From $\frac{1}{u_2} - \frac{1}{350} = \frac{1}{80}$ we get $u_2 = 65.12 \approx 65$ mm.

c The overall magnification is $M = m_o \times m_e \times \frac{D}{v_2} = \left(-\frac{100}{25}\right) \times \left(-\frac{-350}{65.12}\right) \times \frac{250}{350} = -15.4 \approx -15$.

30 See diagram below.



31 The objective forms the first image at a distance v_1 from the objective where $\frac{1}{30} + \frac{1}{v_1} = \frac{1}{24}$ and so $v_1 = 120$ mm. The magnification of the objective is $m_o = -\frac{120}{30} = -4.0$. The overall magnification is

$M = m_o \times m_e = m_o \times \left(1 + \frac{D}{f_e}\right)$ i.e. $-30 = -4.0 \times \left(1 + \frac{250}{f_e}\right)$ and so $1 + \frac{250}{f_e} = 7.5$ giving $f_e = 38.5 \approx 38$ mm.

32 The blue line through the middle of the eyepiece lens is a construction ray.

33 **a** The final image is formed at infinity.

b $M = \frac{f_o}{f_e} \Rightarrow 14 = \frac{2.0}{f_e}$. Hence $f_e = 0.14$ m.

34 **a** The angular width of the moon is $\theta = \frac{3.5\times10^6}{3.8\times10^8} = 0.00921 \approx 0.0092$ rad. (This is

$\theta = 0.00921\times\frac{180^\circ}{\pi} = 0.527^\circ \approx 0.53^\circ$.)

b The angular magnification is $M = \frac{f_o}{f_e} = \frac{3.6}{0.12} = 30$. The diameter of the image of the moon is then

$30\times0.00921 = 0.276 \approx 0.28$ rad.

35 **a** The angular magnification is $M = \frac{f_o}{f_e} = \frac{80}{20} = 4.0$.

b The angle subtended by the building without a telescope is $\theta = \frac{65}{2500} = 0.0260$ rad and so the angle subtended by the image is $\theta' = M\theta = 4\times0.0260 = 0.104$ rad.

36 **a** $M = \frac{f_o}{f_e} = \frac{67}{3.0} = 22.3 \approx 22$.

b $f_o + f_e = 70$ cm

37 The objective focal length must be 57 cm. If the final image is formed at infinity, it means that the image in the objective is formed at a distance of 3.0 cm from the eyepiece i.e. $61.5 - 3.0 = 58.5$ cm from the objective lens.

Hence $\frac{1}{u} + \frac{1}{58.5} = \frac{1}{57.0} \Rightarrow u = 2223 \text{ cm} \approx 22$ m.

38 **a** A technique in radio astronomy in which radio waves emitted by distant sources are observed by an array of radio telescopes which combine the individual signals into one.

b $\theta \approx 1.22\times\frac{\lambda}{b} = 1.22\times\frac{0.21}{25\times10^3} = 1.0\times10^{-5}$ rad.

c The smallest angular separation that can resolved is 1.0×10^{-5} rad. The smallest distance is therefore $2\times10^{22}\times1.0\times10^{-5} = 2\times10^{17}$ m.

39 The universe is full of sources that emit at all parts of the electromagnetic spectrum not just optical light.

40 They do not suffer from spherical aberrations.

41 Advantages: free of atmospheric turbulence and light pollution; no atmosphere to absorb specific wavelengths
Disadvantages: expensive to put in orbit; expensive to service.

C3 Fibre optics

42 $n = \frac{c}{c_m} \Rightarrow c_m = \frac{c}{n} = \frac{3\times10^8}{1.45} = 2.07\times10^8 \text{ m s}^{-1}$.

43 **a** The phenomenon in which a ray approaching the boundary of two media reflects without any refraction taking place.

b Critical angle is that angle of incidence for which the angle of refraction is 90°.

c The critical angle is found from $n_1 \sin\theta_c = n_2 \sin 90^\circ \Rightarrow \sin\theta_c = \frac{n_2}{n_1}$. Since the sine of an angle cannot exceed 1 we must have $n_2 < n_1$ for the critical angle to exist. So total internal reflection is a one way phenomenon.

44 $n_1 \sin\theta_c = n_2 \sin 90^\circ \Rightarrow \sin\theta_c = \frac{n_2}{n_1} = \frac{1.46}{1.50} \Rightarrow \theta_c = 76.7^\circ$.

45 a We know that: $n_1 \sin\theta_C = n_2 \sin 90^\circ$ hence $\sin\theta_C = \dfrac{n_2}{n_1}$.

Then $\cos\theta_C = \sqrt{1-\sin^2\theta_C} = \sqrt{1-\dfrac{n_2^2}{n_1^2}} = \dfrac{\sqrt{n_1^2-n_2^2}}{n_1}$.

b From the diagram, $1 \quad \sin A = n_1 \sin a$. But $a = 90^\circ - \theta_C$ and so $\sin a = \sin(90^\circ - \theta_C) = \cos\theta_C$. Hence

$\sin A = n_1 \cos\theta_C = n_1 \times \dfrac{\sqrt{n_1^2-n_2^2}}{n_1} = \sqrt{n_1^2-n_2^2}$.

c $A = \arcsin\sqrt{n_1^2-n_2^2} = \arcsin\sqrt{1.50^2-1.40^2} = 32.6^\circ$.

46 $A = \arcsin\sqrt{n_1^2-n_2^2} = \arcsin\sqrt{1.52^2-1.44^2} = 29.1^\circ$.

47 We must have $\sqrt{n_1^2-1.42^2} = 1 \Rightarrow n_1 = 1.7367 \approx 1.74$.

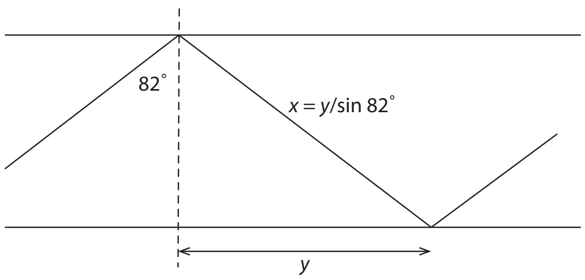
48 It has to be exceptionally pure.

49 a Dispersion is the phenomenon in which the speed of a wave depends on wavelength. This means that the different wavelength components of a beam of light will take different times to travel the same distance.

b Material dispersion is the dispersion discussed in (a). Waveguide dispersion has to do with rays of light following different paths in an optic fibre and hence taking different times to arrive at their destination.

50 a $n = \dfrac{c}{c_m} \Rightarrow c_m = \dfrac{c}{n} = \dfrac{3\times10^8}{1.52} = 1.9737\times10^8 \approx 1.97\times10^8\ \text{m s}^{-1}$.

b The shortest time will be for a ray that travels down the length of the fibre on a straight line of length 8.0 km, i.e. the time of travel will be $\dfrac{8.0\times10^3}{1.9737\times10^8} = 4.05\times10^{-5}$ s. The longest time of travel will be for that ray that undergoes as many internal reflections as possible. The length of the path travelled is then $\dfrac{8.0}{\sin 82^\circ} = 8.0786 \approx 8.08$ km (see diagram) and so the time is $\dfrac{8.0786\times10^3}{1.9737\times10^8} = 4.09\times10^{-5}$ s.



51 The height of the pulses will be less and the width of the pulses greater.

52 a A monomode optic fibre is a fibre with a very thin core so that effectively all rays entering the fibre follow the same path. In a multimode fibre (which is thicker than a monomode fibre) rays follow very many paths of different length in getting to their destination.

b The transition from multimode to monomode fibres offers a very large increase in bandwidth. As discussed also in question 13, dispersion limits the maximum frequency that can be transmitted and hence the bandwidth. A very small diameter monomode fibre will suffer the least from modal dispersion (and hence the distortion and widening of the pulse) and material dispersion is also minimised by using lasers rather than LED's. Hence the bandwidth is increased as the monomode fibre diameter is decreased and laser light is used.

53 Advantages include:

(i) the low attenuation per unit length which means that a signal can travel large distances before amplification
(ii) increased security because the signal can be encrypted and the transmission line itself cannot easily be tampered with
(iii) large bandwidth and so a large information carrying capacity
(iv) not susceptible to noise
(v) they are thin and light and
(vi) do not radiate so there is no crosstalk between lines that are close to each other.

54 The main cause of attenuation in an optic fibre is scattering of light off impurities in the glass making up the core of the fibre.

55 Let P_{in} be the power in to the first amplifier. Then the power out of the first amplifier is $P' = P_{in} \times 10^{\frac{G_1}{10}}$. This is input to the second amplifier so its output is $P_{out} = \left(P_{in} \times 10^{\frac{G_1}{10}}\right) \times 10^{\frac{G_2}{10}} = P_{in} \times 10^{\frac{G_1}{10}+\frac{G_2}{10}} = P_{in} \times 10^{\frac{G_1+G_2}{10}}$ showing that the gain overall is $G_1 + G_2$.

56 The power loss is $10\log\frac{P_{out}}{P_{in}} = 10\log\frac{3.20}{4.60} = -1.58$ dB.

57 The power loss is $10\log\frac{P_{out}}{P_{in}} = 10\log\frac{5.10}{8.40} = -2.167$ dB. So the loss per km is $\frac{2.167}{25} = 0.087\ \text{dB km}^{-1}$.

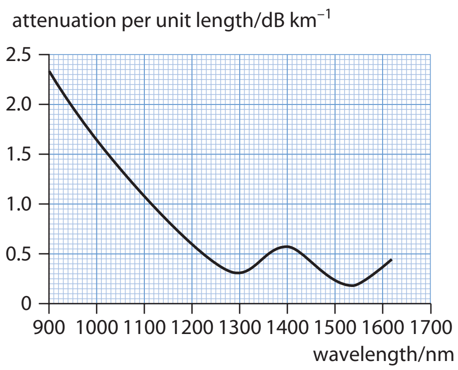
58 The power loss when the power falls to 70% of the original input power is $10\log\frac{P_{out}}{P_{in}} = 10\log\frac{0.70P}{P} = -1.55$ dB. So, $12 \times L = 1.55 \Rightarrow L = 0.13$ km.

59 There is no overall gain in power since $+15 - 12 = 3.0$ dB. Let the input power be P. Then the output power is $P' = P \times 10^{\frac{G}{10}} = P \times 10^{0.3} \approx 2.0P$.

60 There is no overall gain or loss in power since $+7 - 10 + 3 = 0$ dB. So the output power is the same as the input power, the ratio is 1.

61 The overall gain is $10\log\frac{P_{out}}{P_{in}} = 10\log\frac{2P}{P} = 10\log 2 \approx 3.0$ dB. Hence $-12 + G - 6.0 = 3.0$ dB giving $G = 21$ dB.

62 **a** See graph.



b The attenuation per unit length is least for long wavelengths, in particular 1310 nm and 1550 nm, and these are infrared wavelengths.

C4 Medical Imaging

63 **a** Attenuation is the loss of energy in a beam as it travels through a medium.

b For X-rays the main mechanism for attenuation is the photoelectric effect in which X-ray photons knock electrons off the medium's atoms, losing energy in the process. This effect is dependent on the medium atoms' atomic number Z. This means that media with different Z have different attenuation which allows an image of the boundary of the two media to be made.

64 The image can be formed faster by using intensifying screen. This is done by having X-rays that have gone through the patient strike a screen containing fluorescent crystals which then emit visible light. The visible light helps form the image on photographic film faster.

65 If neighbouring organs have the same atomic number the boundary of the organs will not appear clearly. By having the patient swallow a barium meal, the atomic number of organs such as the stomach or the intestine is now greater and can be distinguished from its surrounding tissue.

66 The blurry images are caused by X-rays that have scattered in the patient's body and thus now deviate from their original directions. This may be minimised by placing lead strips between the patient and the film, along the direction of the incident X-ray beam. In this way cattred X-rays will be blocked by the strip and not fall on the film.

67 **a** For the top curve the HVT is 6.0 mm and for the other it is about 4.0 mm.

b The larger energy corresponds to the curve with the longer HVT.

68 **a** The HVT is about 5.0 mm and so the liner attenuation coefficient is about $\frac{\ln 2}{5.0} = 0.139 \approx 0.14\ \text{mm}^{-1}$.

b The transmitted intensity must be 20% of the incident and from the graph this corresponds to a length of about 11.5 mm.

69 $0.60 = e^{-\mu \times 4}$ and so $\mu = -\frac{1}{4}\ln 0.6 = 0.128\ \text{mm}^{-1}$. Then, $0.20 = e^{-\mu x}$ and so

$x = -\frac{1}{\mu}\ln 0.2 = -\frac{1}{0.128}\ln 0.2 = 12.6 \approx 13\ \text{mm}$.

70 $\mu = \frac{1}{3}\ln 2 = 0.231\ \text{mm}^{-1}$ and so $I = I_0 e^{-0.231\times 1} = 0.794 I_0 \approx 0.8 I_0$.

71 It means that as the beam moves through the metal the proportion of the total energy of the X-rays carried by high energy photons increases. This is because the low energy photons get absorbed leaving only the high energy photons move through. For the 20 keV photons the transmitted intensity is $I_{20} = I_0 e^{-\frac{\ln 2}{2.2}\times 5} = 0.207 I_0$. For the 25 keV photons it is $I_{25} = I_0 e^{-\frac{\ln 2}{2.8}\times 5} = 0.290 I_0$. Hence $\frac{I_{25}}{I_{20}} = \frac{0.290 I_0}{0.207 I_0} = 1.4$.

72 Ultrasound is sound of frequency higher than about 20 kHz. It is produced by applying an alternating voltage to certain crystals which vibrate as a result emitting ultrasound.

73 The wavelength of this ultrasound is $\lambda = \frac{v}{f} = \frac{1540}{5\times 10^6} = 3.1\times 10^{-4}\ \text{m} = 0.3\ \text{mm}$. The order of magnitude of the size that can be resolved is of the order of the wavelength and so about 0.3 mm.

74 **a** Impedance is the product of the density of a medium and the speed of sound in that medium.

b $Z = \rho c \Rightarrow c = \frac{Z}{\rho} = \frac{1.4\times 10^6}{940} = 1.5\times 10^3\ \text{m s}^{-1}$.

75 **a** The fraction of the transmitted intensity is given by $\frac{4Z_1 Z_2}{(Z_1 + Z_2)^2}$ and in this case equals $\frac{4\times 420\times 1.6\times 10^6}{(420 + 1.6\times 10^6)^2} \approx 1.0\times 10^{-3}$.

b This is a very small fraction of the incident intensity and not enough to be useful for diagnostic purposes. More intensity has to be transmitted which is why the gel like substance is put in between the skin and the transducer; the gel has an intensity closer to the tissue's so more intensity gets transmitted.

76 In the A the signal strength may be converted to a dot whose brightness is proportional to the signal strength. We now imagine a series of transducers along an area of the body. Putting together the images (as dots) of each transducer forms a **two-dimensional image** of the surfaces that cause reflection of the ultrasound pulses. This creates a B scan.

77 The difference in energy for spin up and down states depends on the magnetic field strength; only those protons in regions where the magnetic field has the "right" value will be absorbed and so it is possible to determine where these photons have been emitted from. This is achieved by exposing the patient to an additional non-uniform field, the gradient field.

78 In this imaging technique, the patient is not exposed to any harmful radiation.

The method is based on the fact that protons have a property called spin. The proton's spin will align parallel or anti-parallel to a magnetic field and the energy of the proton will depend on whether its spin is **up** (i.e. parallel to the magnetic field) or **down** (opposite to the field). The state with spin up has a lower energy than that of spin down. The **difference** in energy depends on the magnetic field strength.

The patient is placed in an enclosure that creates a very **uniform** magnetic field throughout the body. A source of radio frequency forces protons with spin up to make a transition to a state with spin down. As soon as this happens the protons will make a transition back down to the spin up state emitting a photon in the process.

The idea is to detect these photons and correlate them with the point from which they were emitted. This is done with the help of a gradient field as discussed in the previous question.